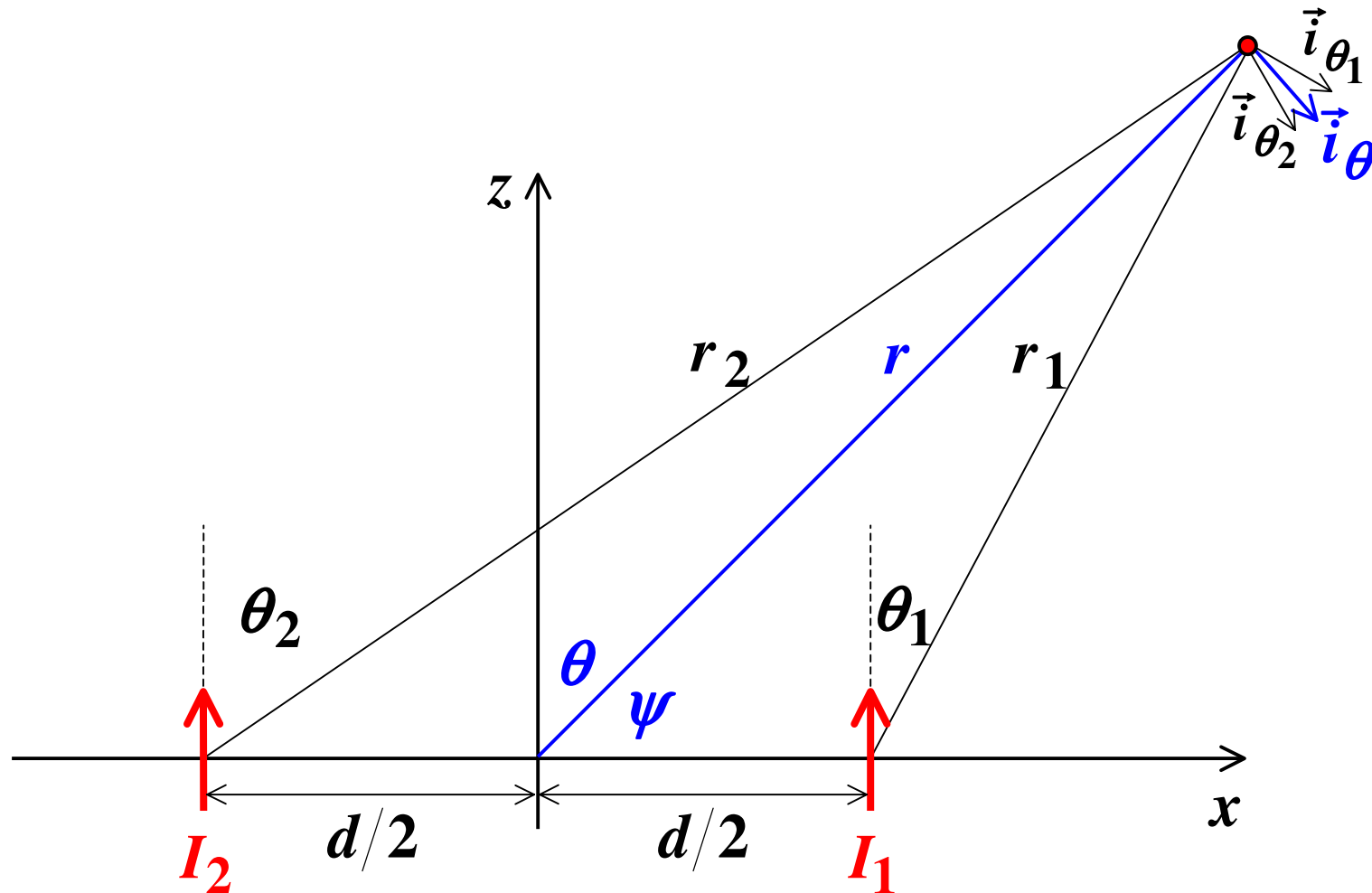


Simple array of two antennas

Consider **two** identical parallel **Hertzian dipoles** separated by a distance d



The **dipole currents** have the **same amplitude** and total **phase difference** α

$$I_1(t) = I_o \cos(\omega t + \alpha/2) \quad \underset{\substack{\Rightarrow \\ \text{phasor}}}{\quad} \quad I_1 = I_o e^{j\alpha/2}$$

$$I_2(t) = I_o \cos(\omega t - \alpha/2) \quad \underset{\substack{\Rightarrow \\ \text{phasor}}}{\quad} \quad I_2 = I_o e^{-j\alpha/2}$$

The **electric far-field** components at the observation point are

$$\vec{E}_1 \approx \vec{i}_{\theta_1} \sqrt{\frac{\mu}{\epsilon}} \frac{j\beta I_o \Delta z e^{-j\beta r_1 + j\alpha/2}}{4\pi r_1} \sin \theta_1$$

$$\vec{E}_2 \approx \vec{i}_{\theta_2} \sqrt{\frac{\mu}{\epsilon}} \frac{j\beta I_o \Delta z e^{-j\beta r_2 - j\alpha/2}}{4\pi r_2} \sin \theta_2$$

At **long distance**, we have

$$r \gg d$$

$$\theta_1 \approx \theta_2 \approx \theta$$

$$\vec{i}_{\theta_1} \approx \vec{i}_{\theta_2} \approx \vec{i}_{\theta}$$

$$r_1 \approx r - \frac{d}{2} \cos \psi \qquad r_2 \approx r + \frac{d}{2} \cos \psi$$

and the **field** components can be written as

$$\vec{E}_1 \approx \vec{i}_{\theta} \sqrt{\frac{\mu}{\epsilon}} \frac{j\beta I_0 \Delta z e^{-j\beta(r - (d/2)\cos\psi) + j\alpha/2}}{4\pi(r - (d/2)\cos\psi)} \sin \theta$$

$$\vec{E}_2 \approx \vec{i}_{\theta} \sqrt{\frac{\mu}{\epsilon}} \frac{j\beta I_0 \Delta z e^{-j\beta(r + (d/2)\cos\psi) - j\alpha/2}}{4\pi(r + (d/2)\cos\psi)} \sin \theta$$

After applying the approximations, the two components can be combined to give the **total electric field**

$$\vec{\mathbf{E}} = \vec{\mathbf{E}}_1 + \vec{\mathbf{E}}_2 \approx \vec{i}_\theta \sqrt{\frac{\mu}{\epsilon}} \frac{j \beta I_0 \Delta z \sin \theta}{4 \pi r} e^{-j \beta r} \times \left(e^{j(\beta(d/2) \cos \psi + \alpha/2)} + e^{-j(\beta(d/2) \cos \psi + \alpha/2)} \right)$$

The final result is

$$\vec{\mathbf{E}} \approx \underbrace{\vec{i}_\theta \sqrt{\frac{\mu}{\epsilon}} \frac{j \beta I_0 \Delta z \sin \theta}{4 \pi r} e^{-j \beta r}}_{\text{field of a Hertzian dipole located at the center of the array}} \underbrace{2 \cos \left(\frac{\beta d \cos \psi + \alpha}{2} \right)}_{\text{array factor}}$$

The resultant radiation pattern of the **electric field** is **proportional** to

$$\underbrace{\sin \theta}_{\text{unit pattern}} \times \underbrace{\cos \left(\frac{\beta d \cos \psi + \alpha}{2} \right)}_{\text{group pattern}}$$

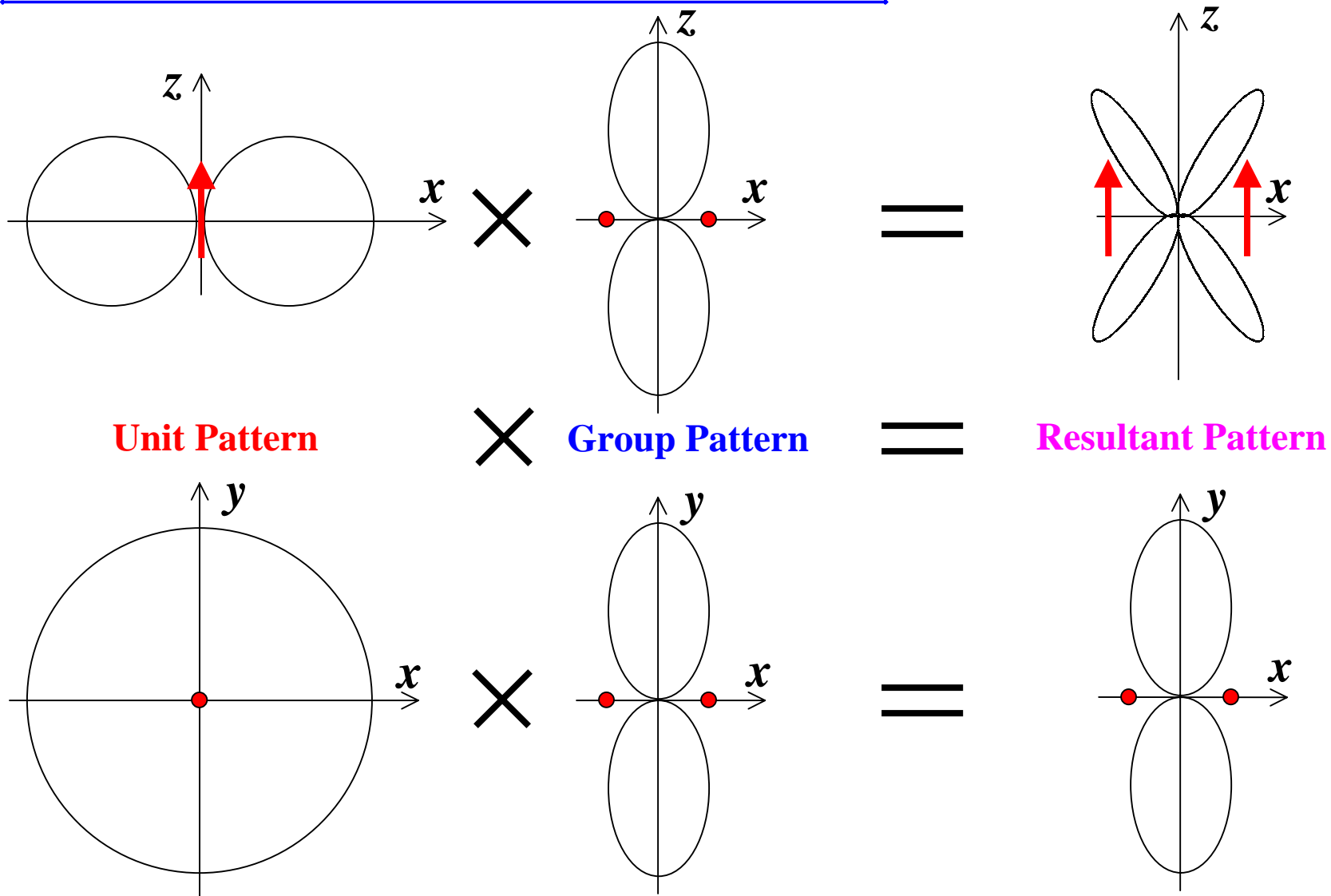
The **unit pattern** is proportional to the radiation pattern of the individual antennas, assumed to be identical.

The **group pattern** is proportional to the radiation pattern the array would have with **isotropic antennas**.

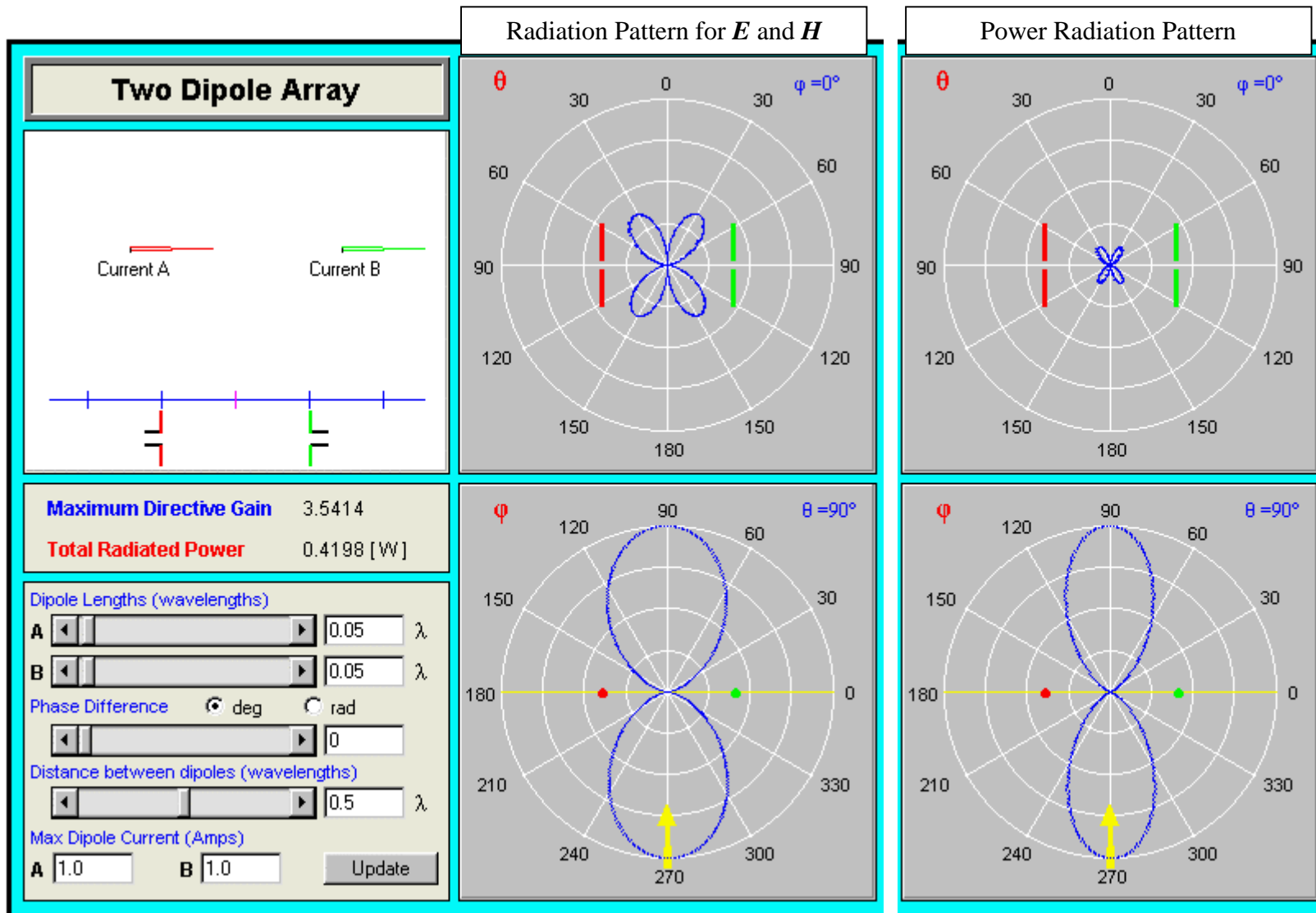
- Note: on the x – y plane, ψ coincides with the azimuthal angle ϕ .

Following are examples of two–antenna arrays with specific values of **dipole distance** and **current phase difference**.

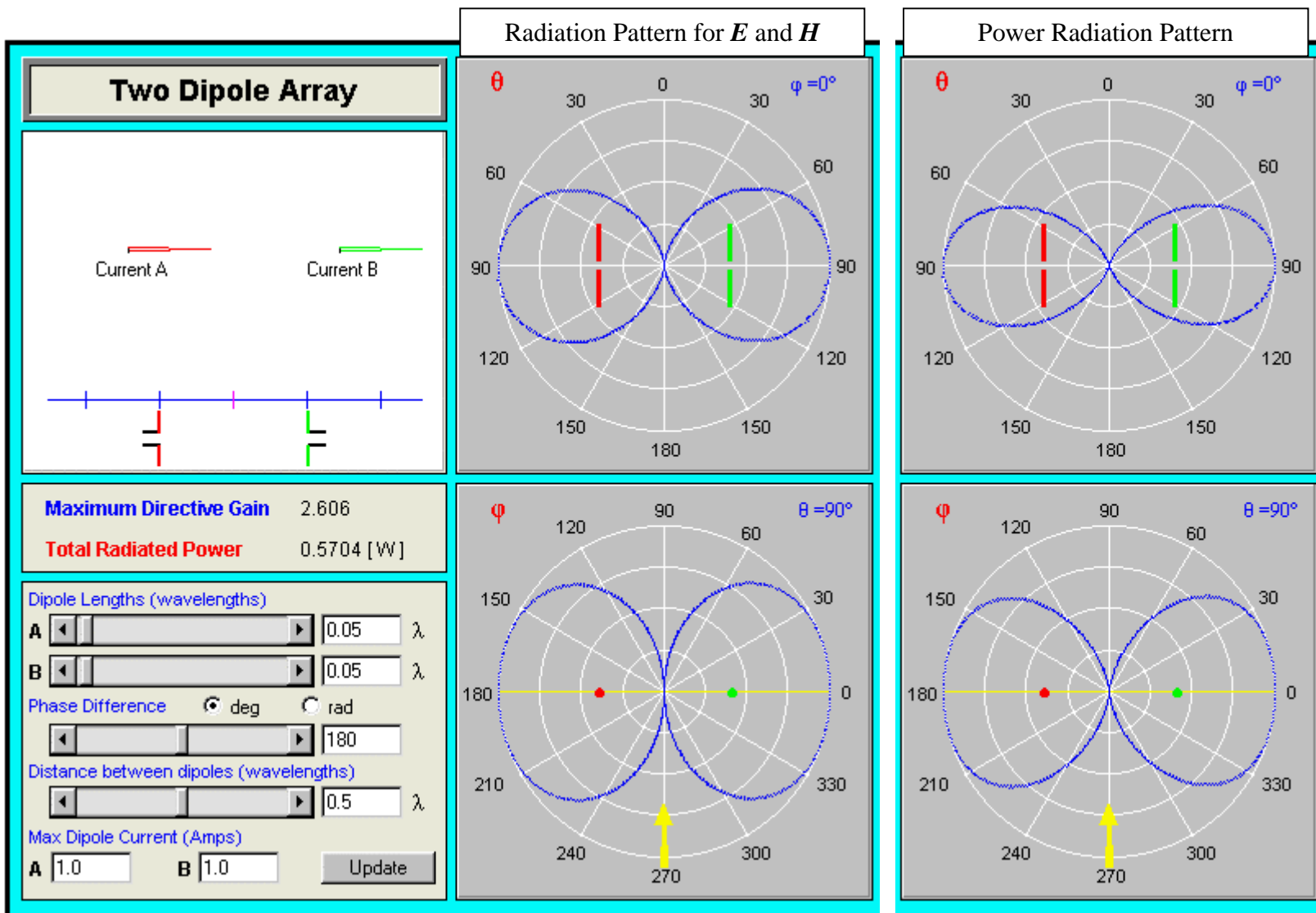
$d = \lambda / 2$ $\alpha = 0$ **Broad-side pattern**



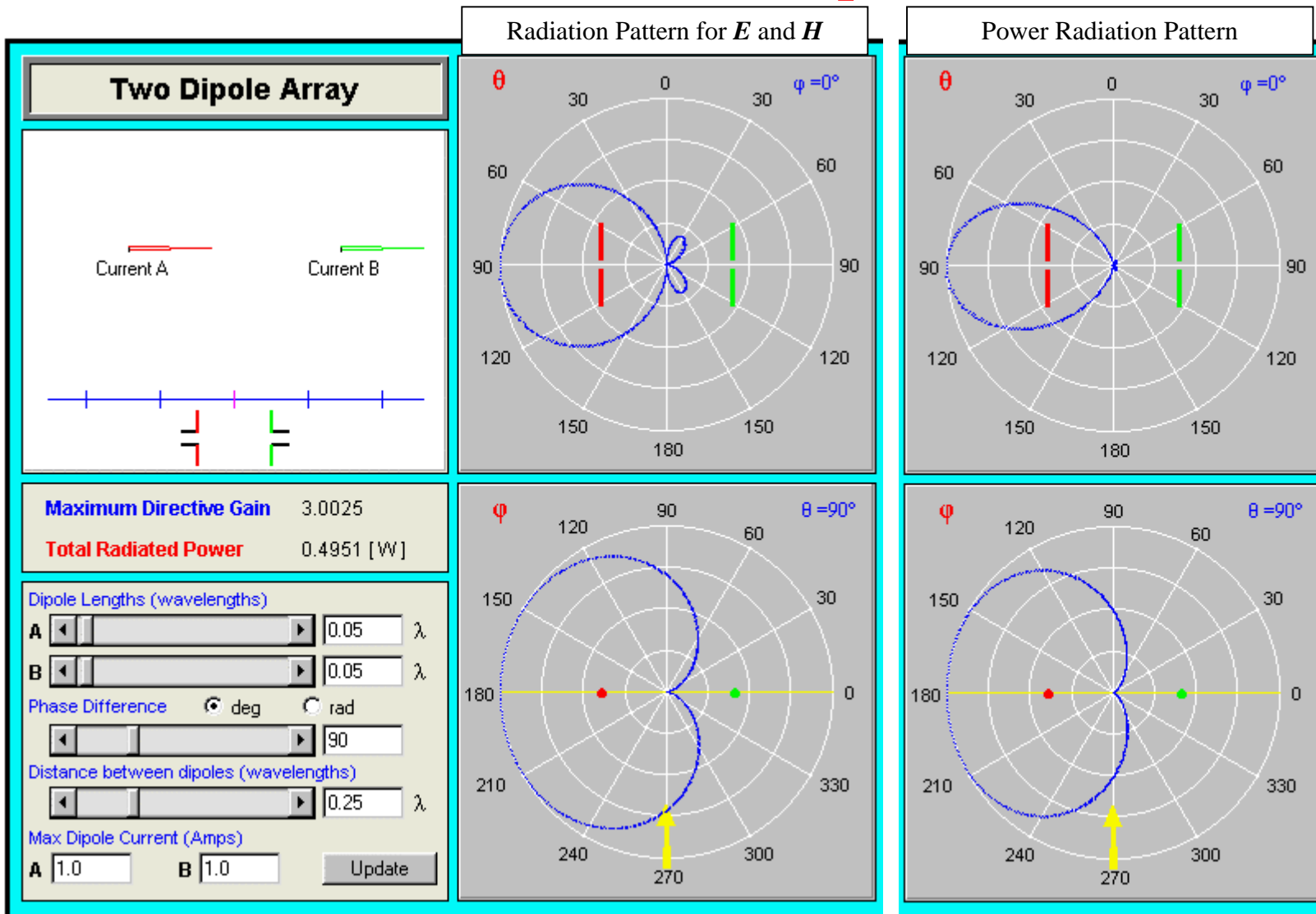
$d = \lambda/2$ $\alpha = 0$ **Broad-side pattern**



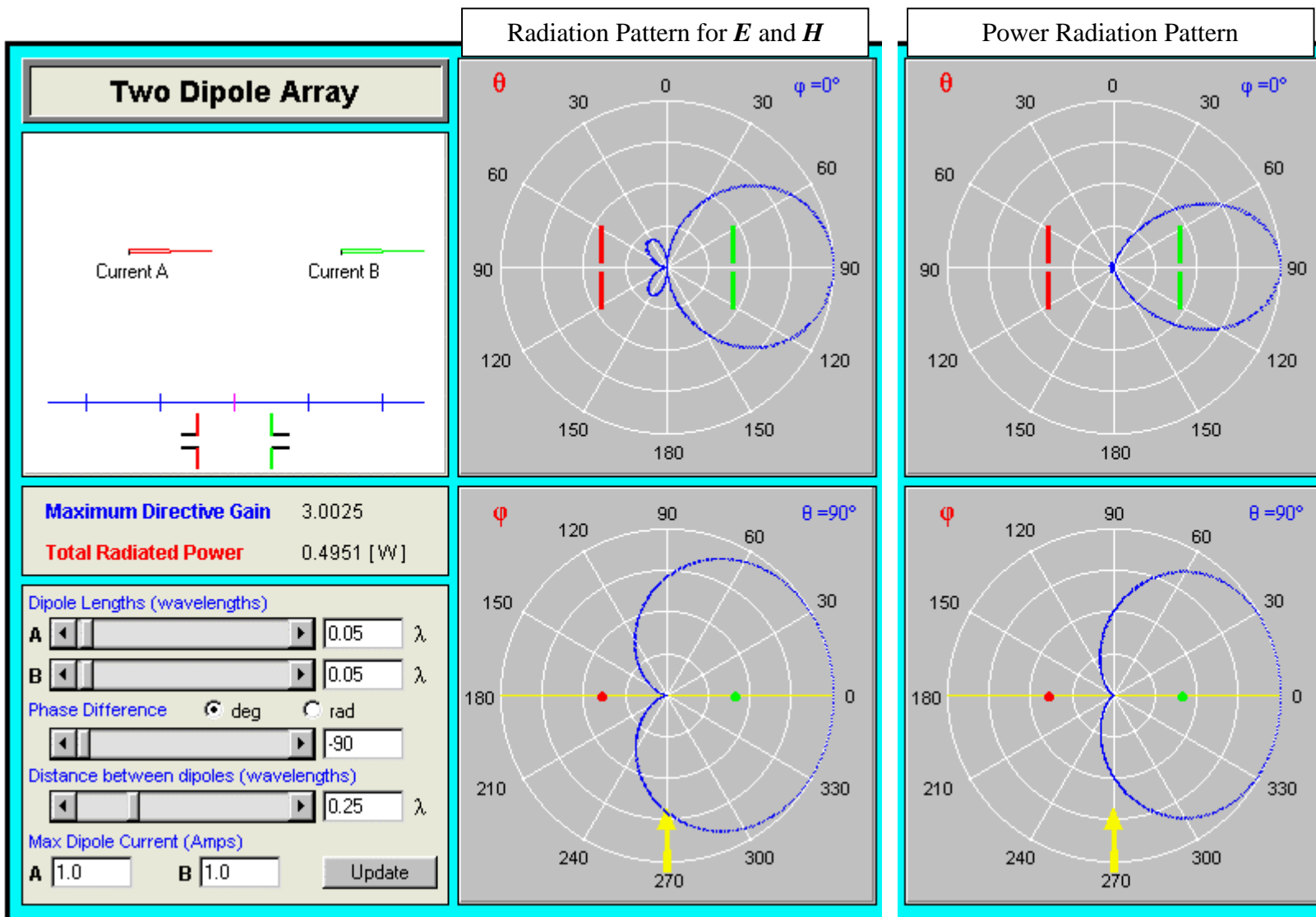
$d = \lambda / 2$ $\alpha = 180^\circ$ **End-fire pattern**



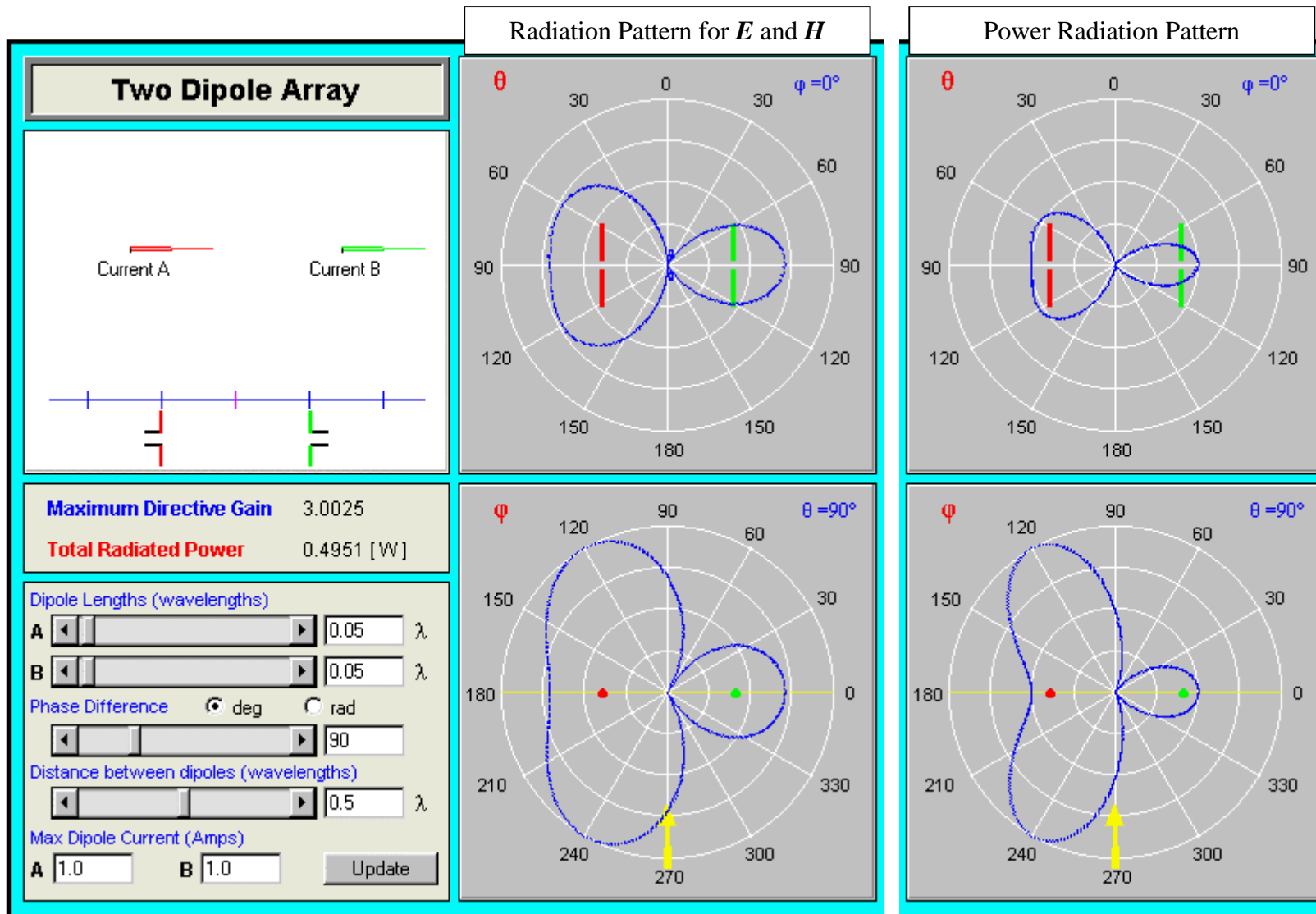
$d = \lambda / 4$ $\alpha = 90^\circ$ **Cardioid pattern**



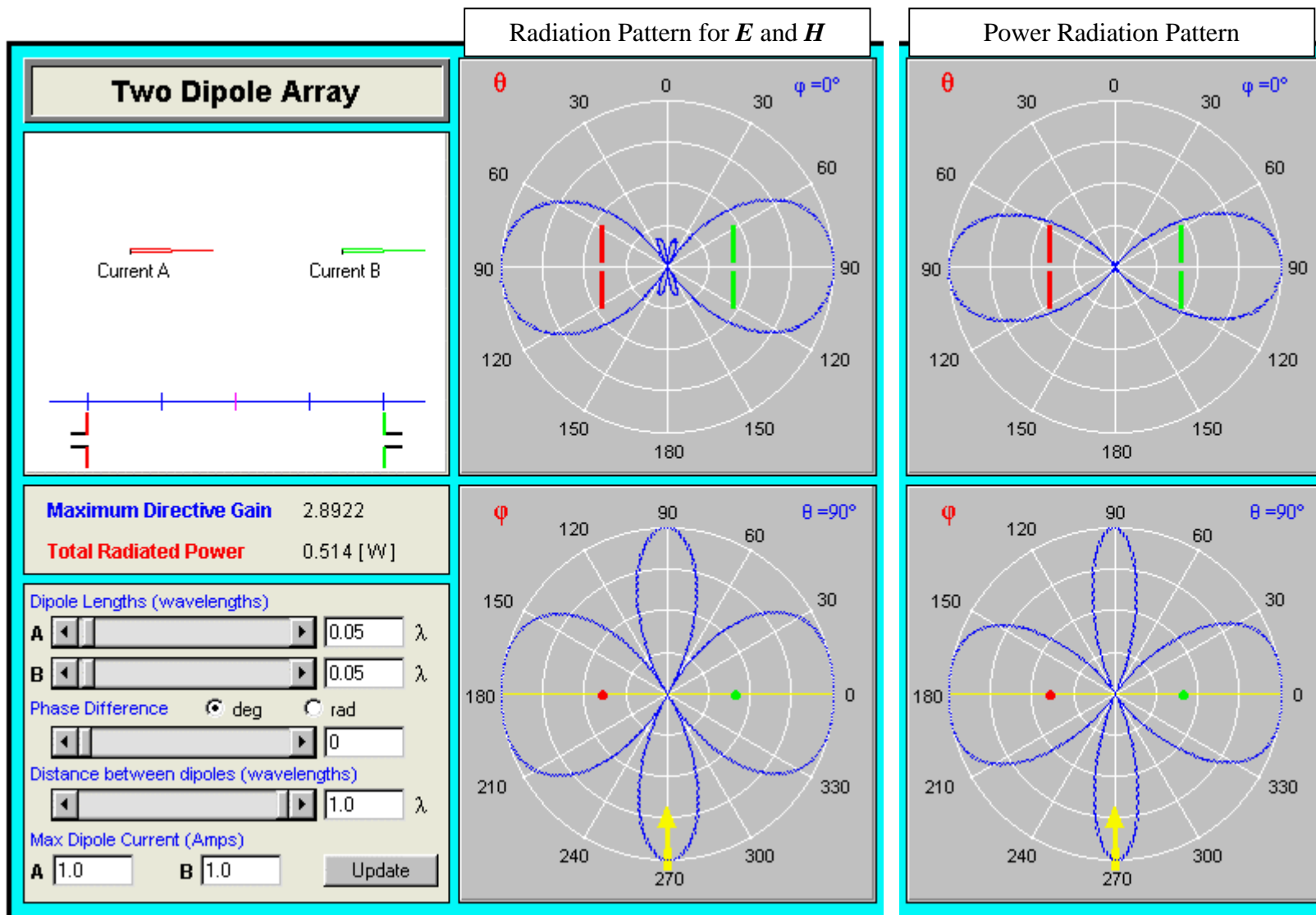
$d = \lambda / 4$ $\alpha = -90^\circ$ **Cardioid pattern**



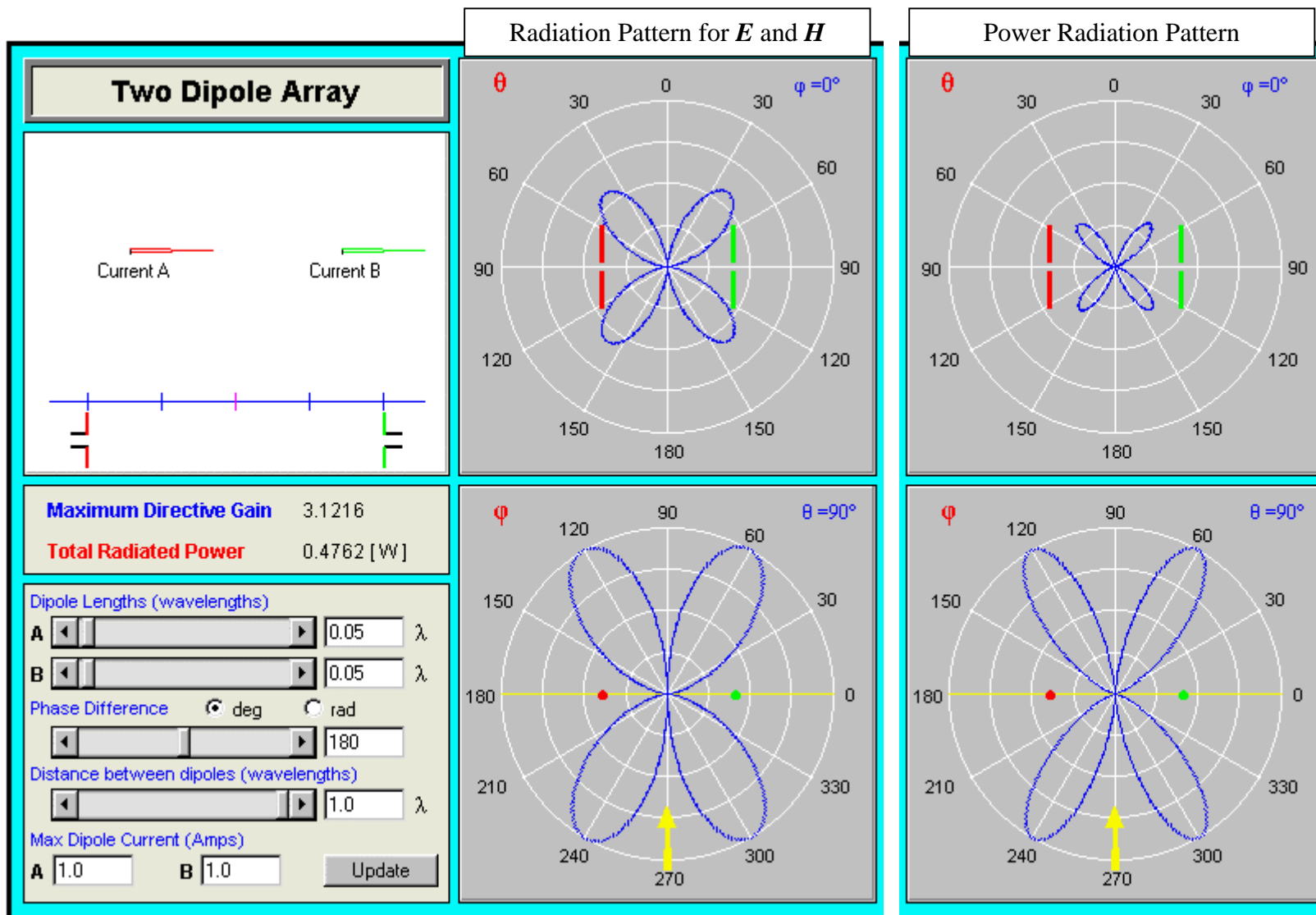
$$d = \lambda / 2 \quad \alpha = 90^\circ$$



$$d = \lambda \quad \alpha = 0^\circ$$



$$d = \lambda \quad \alpha = 180^\circ$$



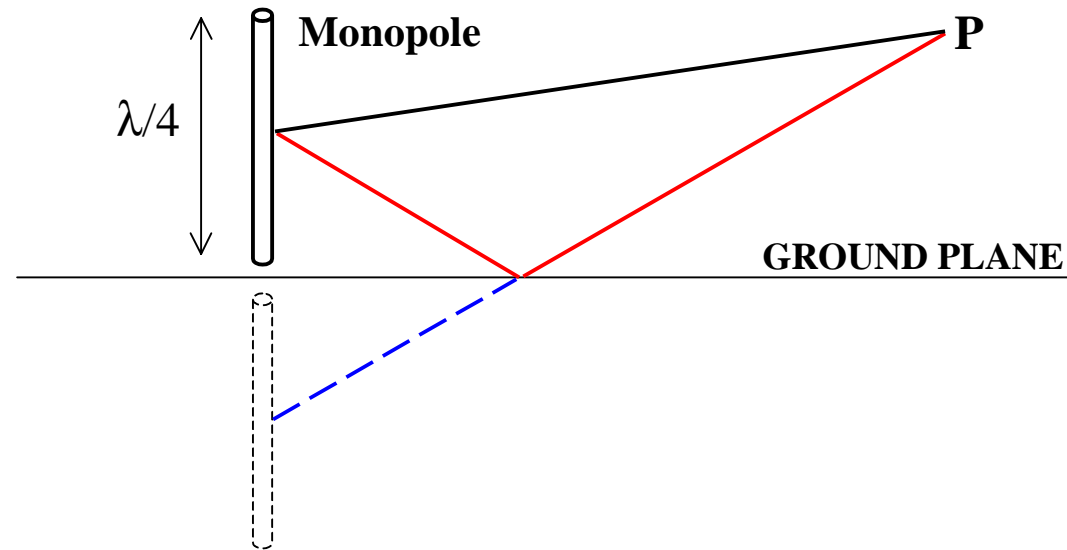
CASE STUDY - 1) A radio broadcast transmitter is located 15 km West of the city it needs to serve. The FCC standard is to have 25 mV/m electric field strength in the city. How much radiation power must be provided to a quarter wavelength monopole?

We consider $\theta=90^\circ$ for transmission in the plane perpendicular to the antenna

$$\vec{E} = \hat{i}_\theta \sqrt{\frac{\mu}{\epsilon}} \frac{j e^{-j\beta r}}{2\pi r} \frac{I_{\max}}{\sin 90^\circ} \cos\left(\frac{\pi \cos 90^\circ}{2}\right) \quad \lambda/2 \text{ dipole}$$

$$|\vec{E}| = 120\pi \frac{I_{\max}}{2\pi \times 15,000} = 0.025 \text{ V/m} \quad \Rightarrow \quad I_{\max} = 6.25 \text{ A}$$

In a **monopole**, the lower wire is substituted by the **ground**. The equivalent radiation resistance is **half** that of the corresponding dipole. Therefore, the total radiated power is half the power radiated by the half-wavelength dipole, for the same current.



A **perfect ground** would act like a **metal surface**, reflecting 100% of the signal. The ground creates an **image** of the “missing” wire delivering to a given point above the ground the same signal as a complete dipole. The transmission line connected to the antenna sees only **half** of the **radiation resistance**, with **total radiated power**:

$$P_{tot} = \frac{1}{2} I_{\max}^2 \underbrace{\sqrt{\frac{\mu}{\epsilon}} \cdot 0.193978 / 2}_{R_{eq}} = 0.5 \cdot 6.25^2 \cdot \frac{73.07}{2} = 713.6 \text{ W}$$

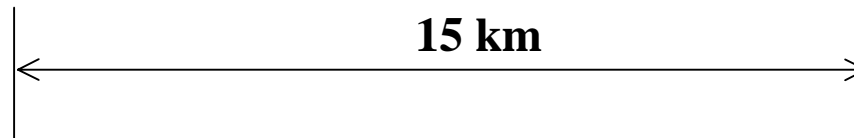
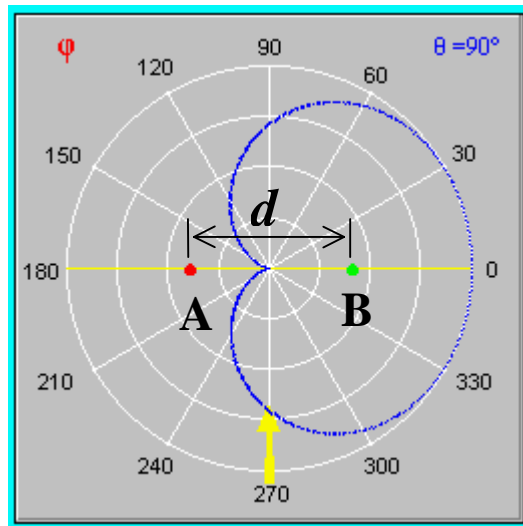
2) Improve the design by using a two-antenna array.

A good choice of array parameters is

$$d = \lambda / 4$$

$$\text{phase}(\text{antenna B}) - \text{phase}(\text{antenna A}) = \alpha = -90^\circ$$

which gives a **cardioid pattern**



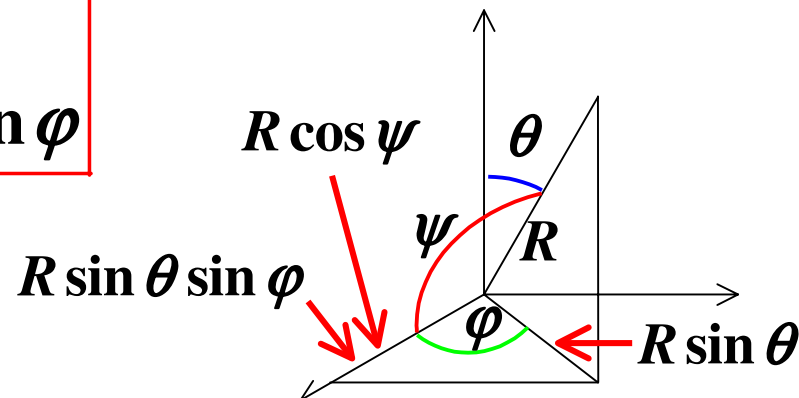
The **Poynting vector** is given by

$$\langle \vec{P}(t) \rangle = \underbrace{\vec{i}_r \sqrt{\frac{\mu}{\epsilon}} \frac{I_{\max}^2}{8\pi^2 r^2 \sin^2 \theta} \cos^2 \left(\frac{\pi \cos \theta}{2} \right)}_{\lambda/2 \text{ dipole Poynting vector}} \underbrace{4 \cos^2 \left(\frac{\beta d \cos \psi + \alpha}{2} \right)}_{(\text{array factor})^2}$$

$$= \vec{i}_r \sqrt{\frac{\mu}{\epsilon}} \frac{I_{\max}^2}{8\pi^2 r^2 \sin^2 \theta} \cos^2 \left(\frac{\pi \cos \theta}{2} \right) \underbrace{4 \cos^2 \left(\frac{\pi}{4} \cos \psi - \frac{\pi}{4} \right)}_{(\text{array factor})^2}$$

Note

$$\cos \psi = \sin \theta \sin \varphi$$



The **total radiated power** is

Only $\pi/2$ because it is a monopole

$$P_{tot} = r^2 \int_0^{\pi/2} \sin \theta d\theta \int_0^{2\pi} \langle \vec{P}(t) \rangle d\varphi$$

$$P_{tot} = r^2 \int_0^{\pi/2} \sqrt{\frac{\mu}{\epsilon}} \frac{I_{\max}^2}{8\pi^2 r^2 \sin^2 \theta} \cos^2 \left(\frac{\pi \cos \theta}{2} \right) \sin \theta d\theta$$

$$\int_0^{2\pi} \underbrace{4 \cos^2 \left(\frac{\pi}{4} \sin \theta \sin \varphi - \frac{\pi}{4} \right)}_{(\text{array factor})^2} d\varphi$$

The integral over the **azimuthal angle** φ gives

$$\begin{aligned}
 & 4 \int_0^{2\pi} \cos^2 \left(\frac{\pi}{4} \sin \theta \sin \varphi - \frac{\pi}{4} \right) d\varphi \\
 &= 4 \int_0^{2\pi} \left(\frac{1}{2} + \frac{1}{2} \cos \left(\frac{\pi}{2} \sin \theta \sin \varphi - \frac{\pi}{2} \right) \right) d\varphi \\
 &= 4 \int_0^{2\pi} \left(\frac{1}{2} + \frac{1}{2} \sin \left(\frac{\pi}{2} \sin \theta \sin \varphi \right) \right) d\varphi = 4\pi
 \end{aligned}$$

For a **monopole**, we only have the integral

$$\int_0^{2\pi} d\varphi = 2\pi$$

In the direction of **maximum**

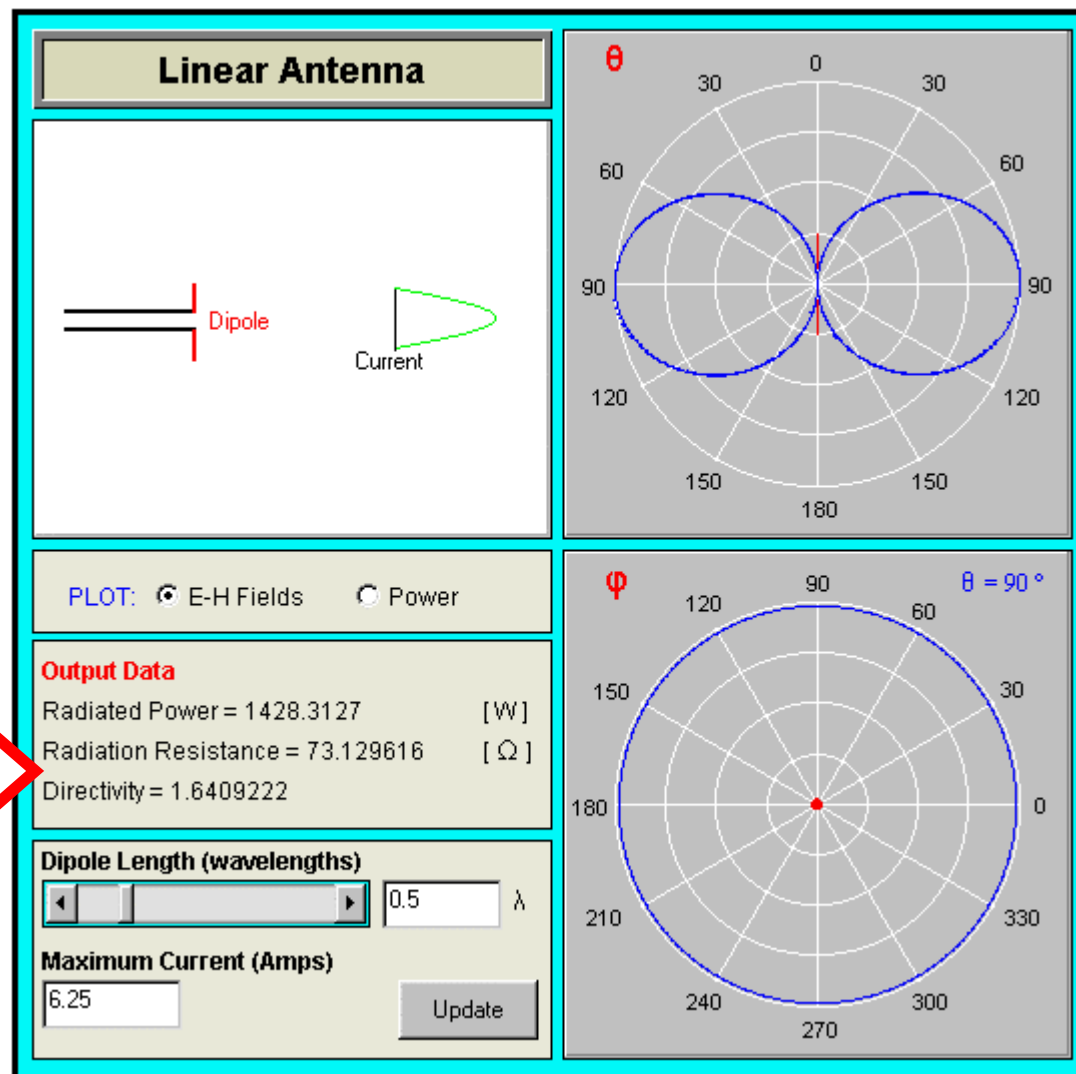
$$\theta = 90^\circ \quad \& \quad \varphi = 90^\circ \Rightarrow \text{array factor} = 2$$

The same **field strength** (25 mV/m) is obtained by applying **half** the **current** of the original monopole to the array elements (also monopoles)

$$I_{\max} = \frac{6.25}{2} = 3.125 \text{ A}$$

The **total radiated power** is proportional to the square of the current, and the integral over φ gives a factor 4π instead of 2π for the array. Overall, the total radiated power needed by the array, to produce the same electric field, is **half** that of the individual monopole

$$P_{\text{tot}} = \frac{714}{2} = 357 \text{ W}$$

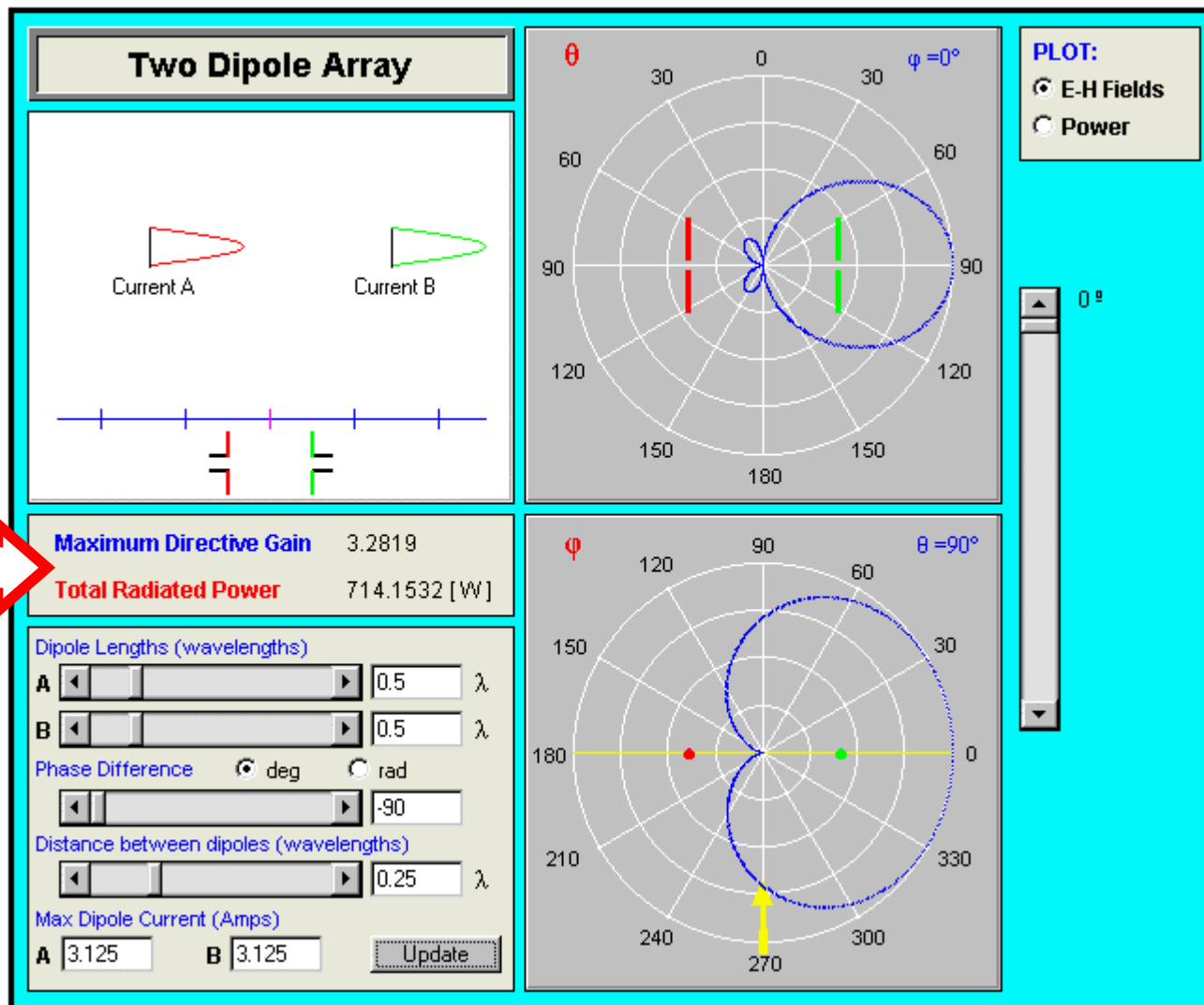


For a monopole

**Radiated Power = 0.5×1428.3127
= 714.155635 [W]**

**Rad. Resistance = 0.5×73.129616
= 36.564808 [Ω]**

**SAME FEEDING CURRENT
FOR BOTH DIPOLE AND
MONOPOLE**

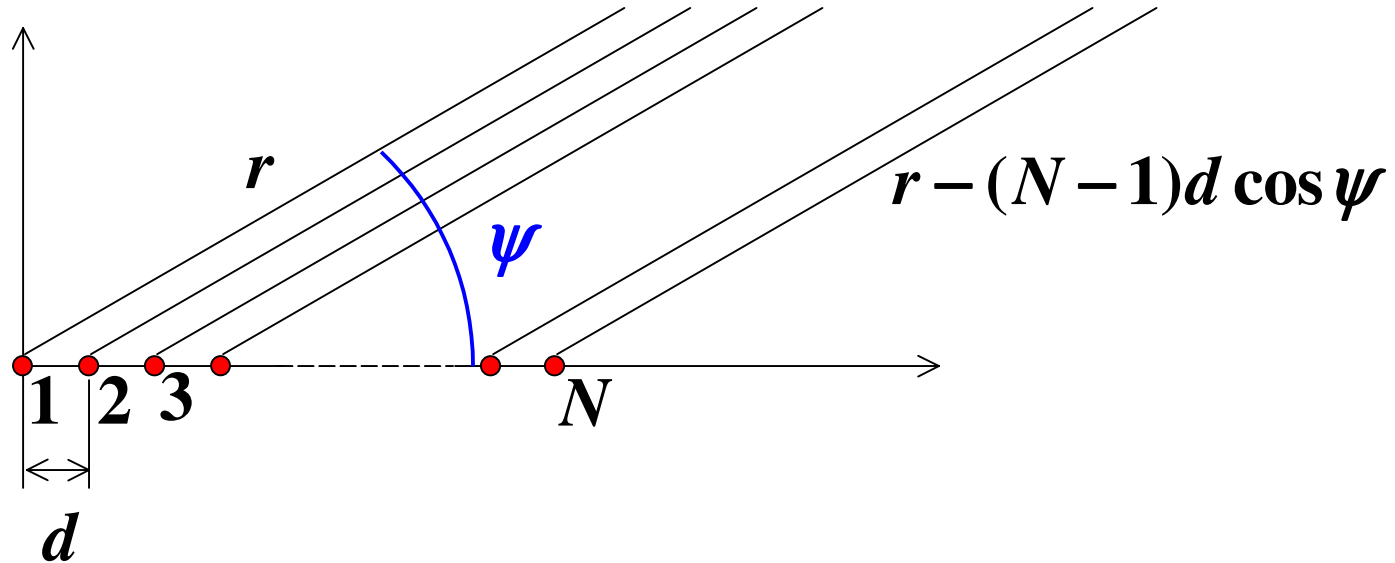


For monopoles
 Radiated Power
 = 0.5×714.1532
 = **307.0766 [W]**

SAME FEEDING CURRENTS FOR BOTH DIPOLES AND MONOPOLES

N-Element Antenna Array

Assume a uniform array of N identical antennas. The elements are fed by currents with constant amplitude and with phase increasing by an amount α from one to the other. The spacing d between the antennas is uniform.



$$I(1) = I_0; \quad I(2) = I_0 e^{j\alpha}; \quad \dots; \quad I(N) = I_0 e^{j(N-1)\alpha}$$

The **electric field** at the observation point (r, ψ) is of the form

$$\begin{aligned}
 \mathbf{E}(r, \psi) &= E_0 e^{-j\beta r} + E_0 e^{-j\beta(r-d \cos \psi)} e^{j\alpha} + \dots \\
 &\quad \dots + E_0 e^{-j\beta(r-(N-1)d \cos \psi)} e^{j(N-1)\alpha} \\
 &= E_0 e^{-j\beta r} \left[1 + e^{j(\beta d \cos \psi + \alpha)} + \right. \\
 &\quad \left. \dots + e^{j(N-1)(\beta d \cos \psi + \alpha)} \right] \\
 &= E_0 e^{-j\beta r} \frac{1 - e^{jN(\beta d \cos \psi + \alpha)}}{1 - e^{j(\beta d \cos \psi + \alpha)}}
 \end{aligned}$$

We have used
$$\sum_{n=0}^{N-1} e^{jn(\beta d \cos \psi + \alpha)} = \frac{1 - e^{jN(\beta d \cos \psi + \alpha)}}{1 - e^{j(\beta d \cos \psi + \alpha)}}$$

The **magnitude** of the **electric field** is given by

$$\begin{aligned}
 |\mathbf{E}(r, \psi)| &= |E_o| \left| \frac{1 - e^{jN(\beta d \cos \psi + \alpha)}}{1 - e^{j(\beta d \cos \psi + \alpha)}} \right| \\
 &= |E_o| \underbrace{\left| \frac{\sin[N(\beta d \cos \psi + \alpha) / 2]}{\sin[(\beta d \cos \psi + \alpha) / 2]} \right|}_{\text{array factor}}
 \end{aligned}$$

We have used

$$\left| 1 - e^{jx} \right| = \left| 2 j \sin \frac{x}{2} e^{jx/2} \right| = 2 \left| \sin \frac{x}{2} \right|$$

We can rewrite

$$|\mathbf{E}(r, \psi)| = N |E_0| \underbrace{\left| \frac{1}{N} \frac{\sin[N(\beta d \cos \psi + \alpha) / 2]}{\sin[(\beta d \cos \psi + \alpha) / 2]} \right|}_{\text{group pattern}}$$

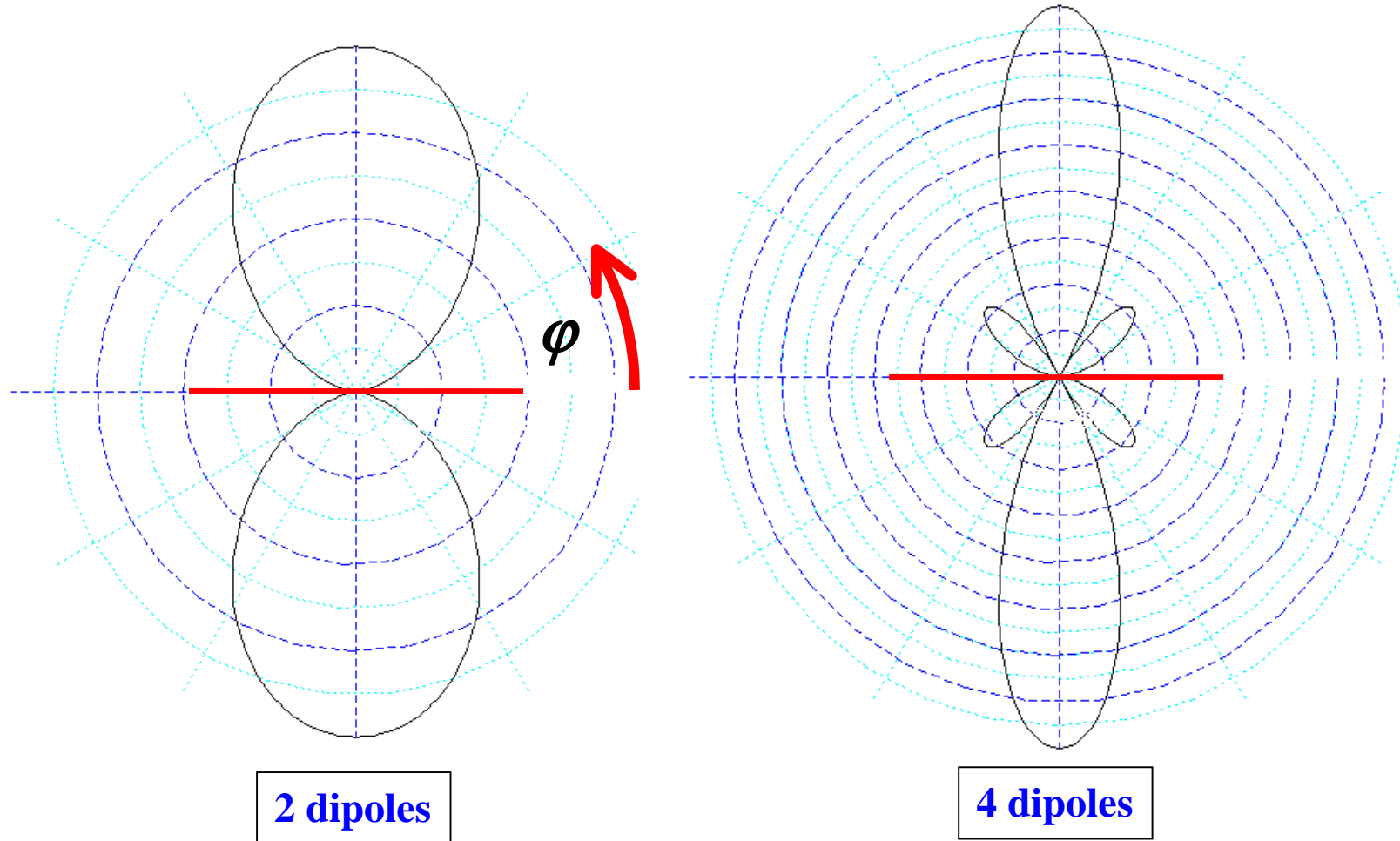
The group pattern has

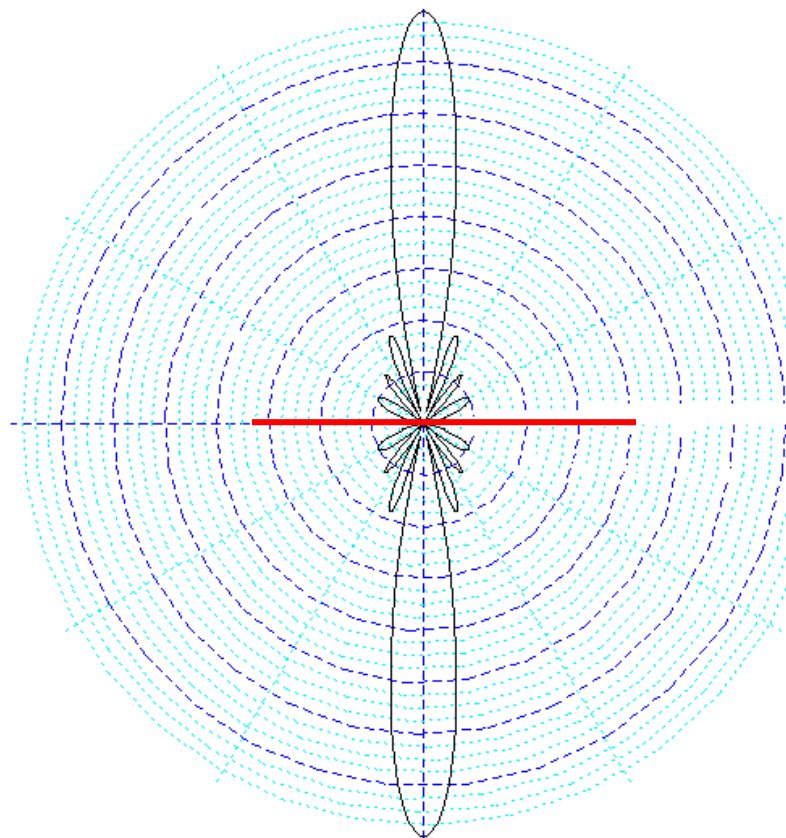
Maxima when $\beta d \cos \psi + \alpha = 0, 2\pi, 4\pi \dots$

Nulls when $N(\beta d \cos \psi + \alpha) = 2m\pi$
for $m = \text{integer} \neq 0, N, 2N, \dots$

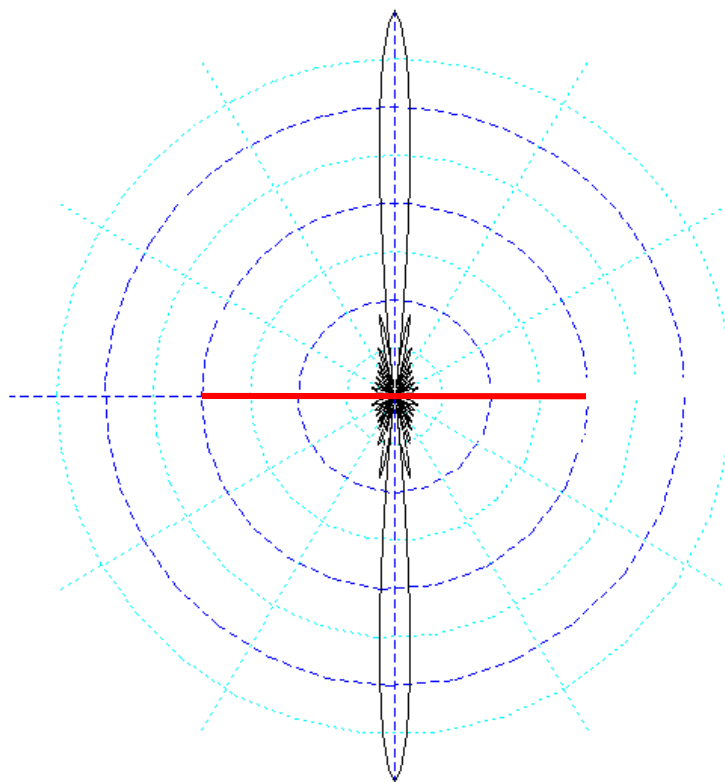
Examples

Broadside array (plots on the azimuthal plane, with $\theta = 90^\circ$)



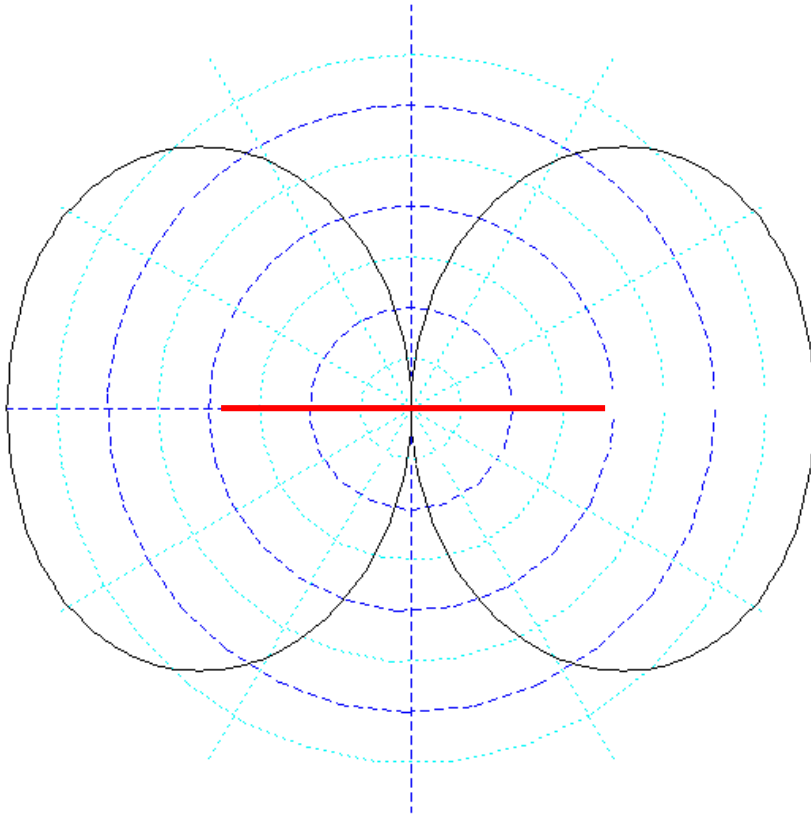


8 dipoles

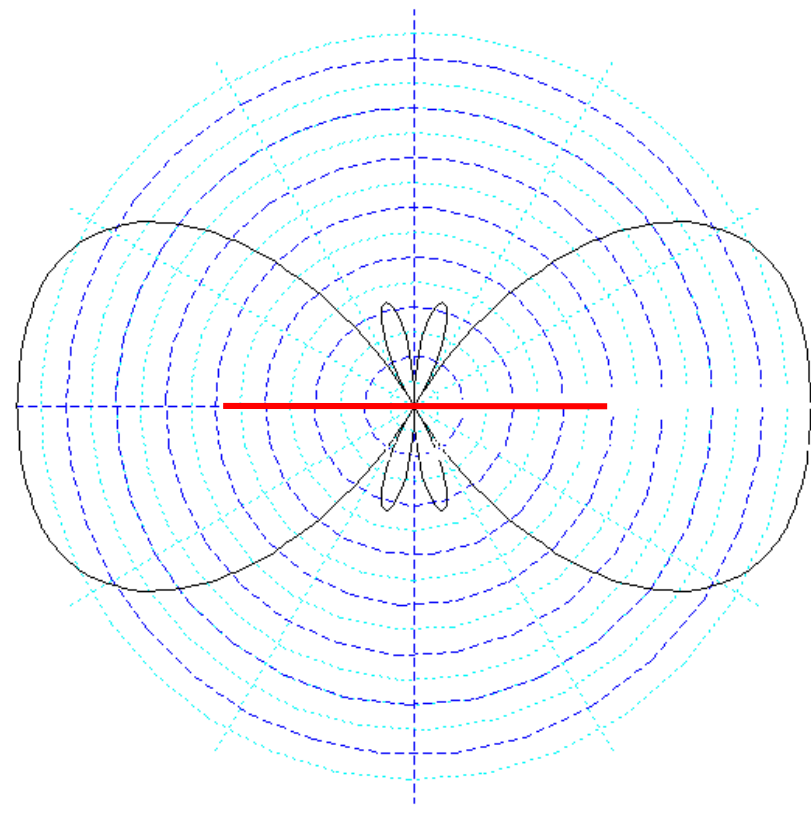


16 dipoles

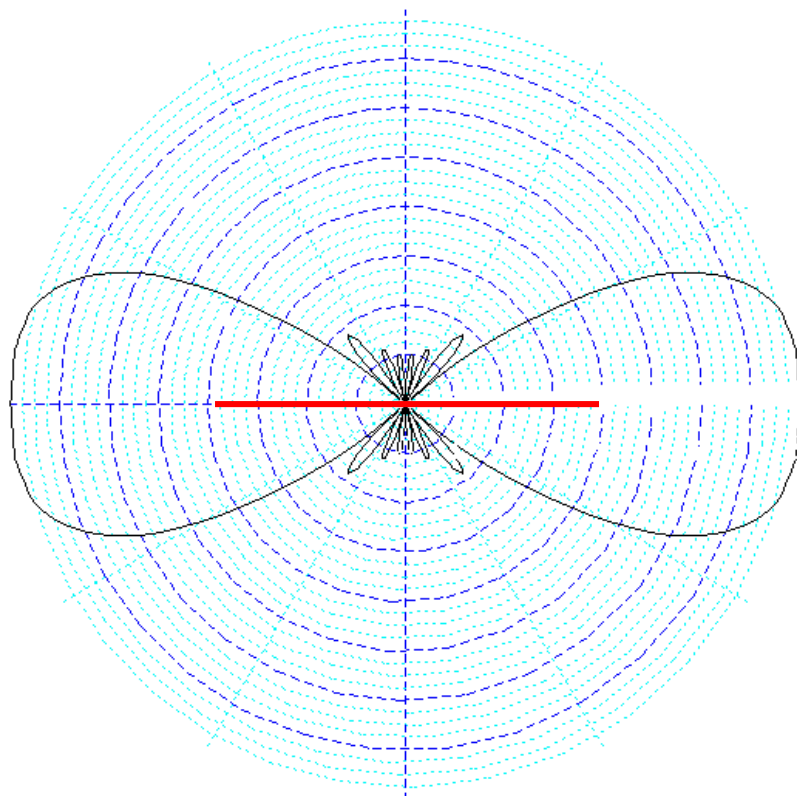
End-fire array (plots on the azimuthal plane, with $\theta = 90^\circ$)



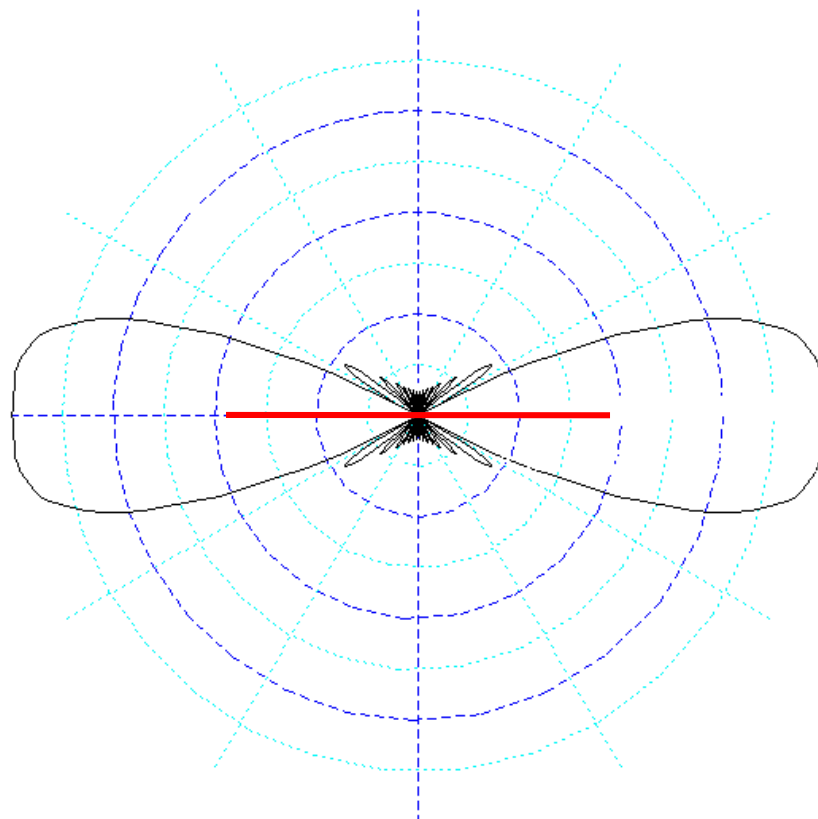
2 dipoles



4 dipoles

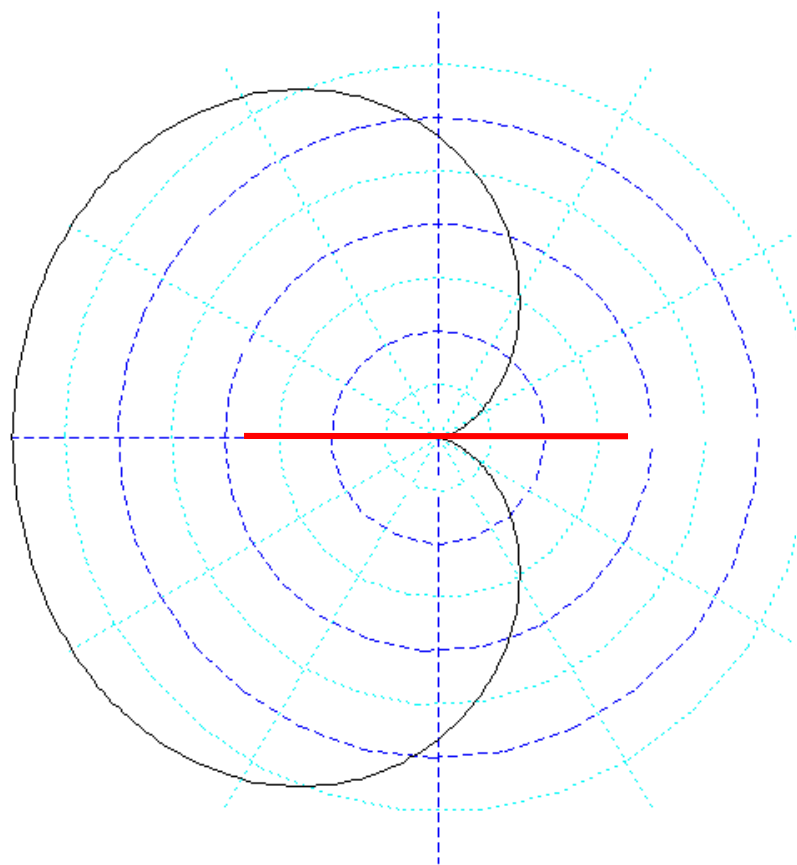


8 dipoles

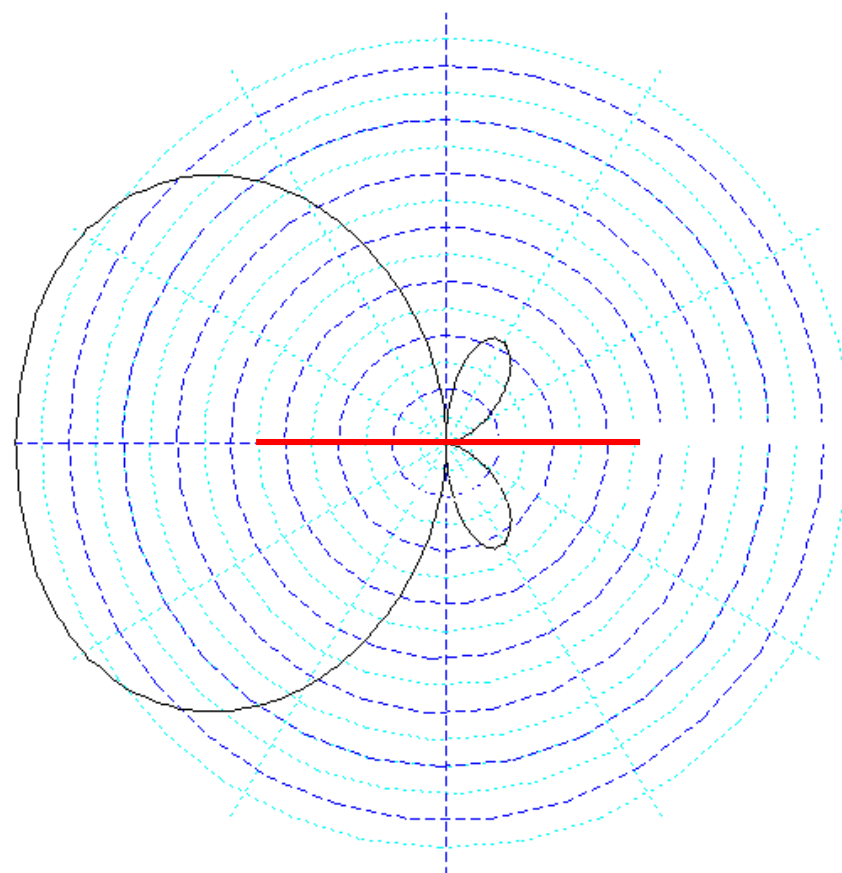


16 dipoles

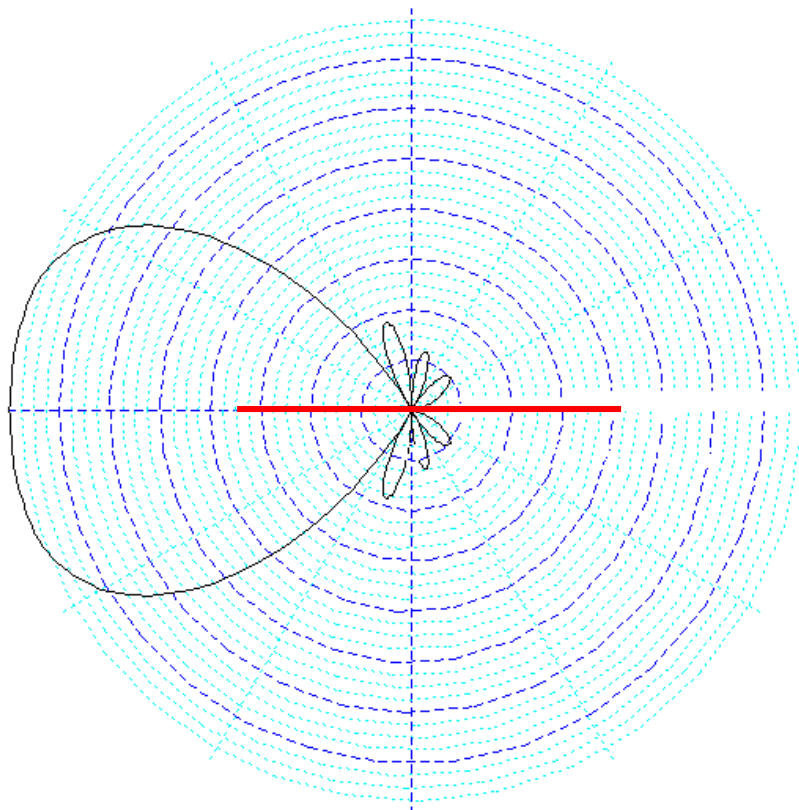
Cardioid array (plots on the azimuthal plane, with $\theta = 90^\circ$)



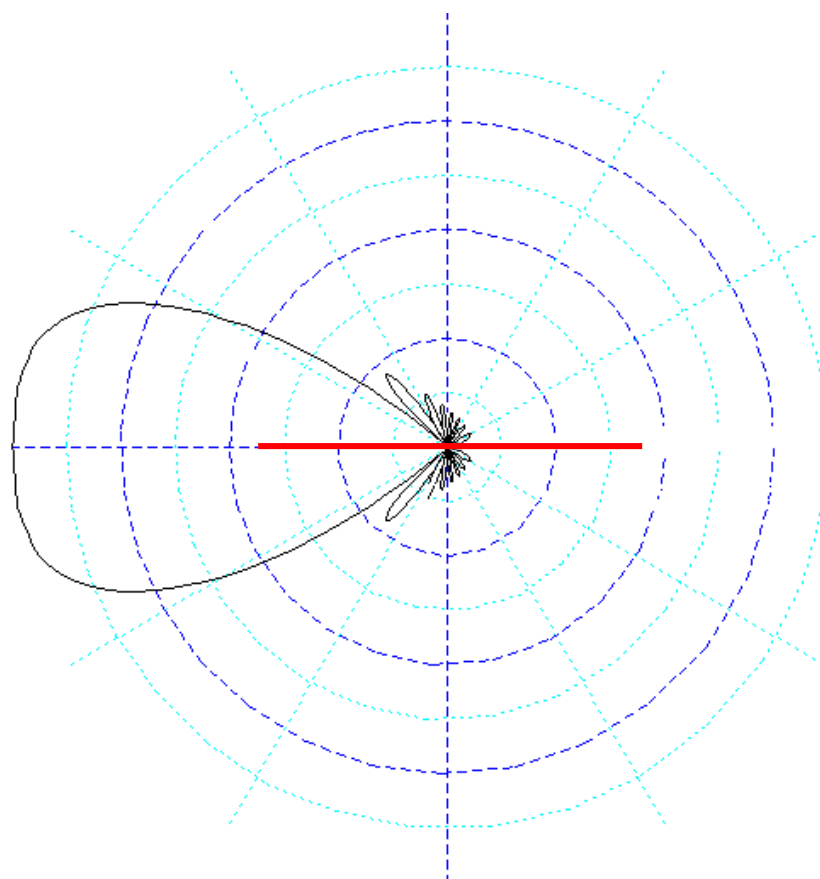
2 dipoles



4 dipoles



8 dipoles



16 dipoles